

Simulating flood recovery manoeuvres using a free running submarine model

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ABSTRACT

The requirement to manage the safety of a submarine necessitates a number of key factors to be understood; one of which is to understand the manoeuvring performance of the submarine throughout the design process. A free running submarine model forms a fundamental component of this predictive capability so QinetiQ's Submarine Research Model (SRM) II was designed to support the UK MOD with their ongoing hydrodynamic assessment and modelling of the manoeuvring and control of the Royal Navy's current and future submarines. The SRMII design is based on a space-frame construction and uses COTS products where possible; the design is reconfigurable with critical components transferable between different hull shapes. Uniquely, the SRMII has been designed to have a large and capable ballast and trim system; a system that is capable of emulating the effects of compartment flooding incidents in submarines and moreover capable of representing the subsequent emergency recovery procedures including some aspects of blowing the main ballast tanks. This paper discusses how the ballast and trim system in the SRMII has been used to obtain model scale submarine manoeuvring data that were not obtainable from previous model tests. The experimental data are then used to validate the mathematical model used for predicting the response of a submarine to a compartment flood.

INTRODUCTION

Today's military submarine can typically only operate in a very restricted portion of the world's ocean depths; the typical collapse depths of submarines from World War II were around 280m [1], far less than the average depths of The World's Oceans. To provide assurance that a submarine design would be capable of operating safely within these rather tight boundaries means quantifying the manoeuvring characteristics early on in the design process; this is crucial in reducing the risks of producing designs that are unsuitable for the environment in which they are expected to operate.

The focus of manoeuvring and control studies is towards understanding the performance of a submarine operating in deep water, see [2] for example, and a number of approaches both numerical and experimental, reflecting the state-of-the-art at the time, can be applied throughout the design process. QinetiQ has an active role in assuring submarine safety using an approach, based on complementary numerical and experimental techniques that have been developed from its knowledge of hydrodynamics gained over a number of years, to provide a validated understanding of the manoeuvring and control performance of a submarine design, [3]. Irrespective of the operational requirements of a new platform, by and large, a submerged submarine will be required to manoeuvre safely in the vertical and horizontal planes, which can be translated, in generic terms, to be able to be able to accurately evaluate the performance of a particular hullform and appendage configuration at the design stage to:

- Determine measures of directional stability;
- Establish the size and power requirements of any control surfaces;
- Design suitable motion control systems;

- Determine that standard manoeuvres meet international maritime regulations or national design guidelines; and
- Demonstrate that the submarine can safely recover, within the boundaries of operational requirements, from the consequence of credible system level failure.

Therefore, a requirement for any nation that operates submarines, with due governance regarding safety, the ability to access validated tools and techniques to underpin timely advice, to accurately understand the behaviour of a submarine and to provide safe operating envelopes of current and future underwater platforms during normal operations and emergency conditions is key. This paper describes one aspect of mathematical model validation, free running model tests, particularly focussing on modelling flood events and the subsequent recovery of the submarine. The outcome of this research will provide the design guidance and validated evaluation toolsets for the benefit of:

- Future projects groups investigating submarine design at a concept level;
- Acquisition design groups carrying out more detailed project definition assessments; and
- In-service support to set safety and operational constraints for the submarine fleet.

HAZARD MITIGATION AND SAFE OPERATOR GUIDANCE

Through wider present-day awareness of safety of personnel, hazard mitigation modelling and the provision of Safe Operating Envelopes (SOEs) to submarine command is now considered as best practice by top-tier navies worldwide. Assuring the safety of submarines is paramount to the work that QinetiQ undertakes by considering the key elements that contribute to safety assurance; including design optimisation, provision of validated safety guidance and the development of robust emergency procedures for implementation by trained operators. This enduring support through the life of the submarine underpins the operational safety case that helps ensure that if an incident were to occur, the submarine would be able to recover safely and return to port.

The provision of this safety advice is based on two major incidents from which operators of submarines must be able to safely recover: a control surface jam at high speed and a flooding incident at low speed. The provision of safety guidance that mitigates the risk to the submarine following such incidents requires a detailed understanding of the behaviour of the submarine, a fully validated prediction capability, an understanding of the submarine systems that are crucial to safe recovery and the presentation of data in a way that is useful, unambiguous and easily understandable. Figure 1 provides an example of a Safe Operating Envelope (SOE) in the format of what is known as a Manoeuvring Limitation Diagram (MLD), in the UK, to provide guidance to Submarine Command, see [4]. This example provides boundaries of safe operation in terms of speed and depth; the slow speed boundaries (Flood Avoid Zone or FAZ) are present as a consequence of a flood and at higher speeds the restrictions are limited to mitigate against the risk of a plane jam (Jam lines). In order to generate such a robust set of curves it is crucial that the response of the submarine following such emergency scenarios is known; the only practicable means of doing that is to have a reliable mathematical model of a manoeuvring submarine.

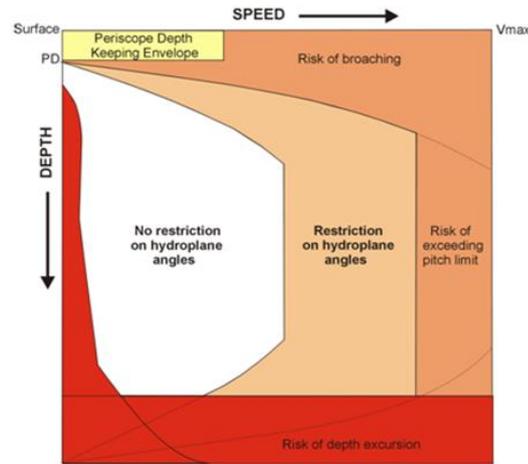


Figure 1: Example of Safe Operator Guidance [4]

The trajectory of a submarine at any instant in time is described in [5] in some detail but the assumptions and simplifications result in a set of 6 simultaneous non-linear equations for small disturbances and slowly varying motion, which are to be solved for the 6 unknown translational and rotational accelerations. Provided the forces and moments on the submarine are known, the accelerations can be found. The time integrations can be then undertaken to determine the translational velocity and rotational rates, and subsequently position and attitude. The key technical challenge is determining the hydrodynamic, and other non-hydrodynamic related, forces and moments on the submarine, at each time step. The essence of the mathematical model of submarine manoeuvring is the determination of the hydrodynamic forces and moments that are acting on the geometry. One mathematical approach assumes that the motion of the submarine is slowly varying [4], and that these quasi-steady state forces and moments on a manoeuvring submarine can be described by a series of empirical equations as described in [6] for example. The approach is then based on establishing a set of hydrodynamic coefficients that relate the state variables of the motion to the 3 forces and 3 moments acting on the submarine; current approaches include physical model tests of a constrained model, numerical methods or a combination of both, see [7] for an expanded explanation of the generic approach and see [8] for this process applied to a particular submarine design. Whether from physical model tests or Computational Fluid Dynamics (CFD), or a combination of both, once the hydrodynamic coefficient set has been obtained, the form of the mathematical model is known and can be used to develop a simulation capability for design studies including investigating the resultant SOE, in lieu of validation activities through free running model tests.

Once a mathematical model is available there are two features of the SOE that require validation through Free Running Model (FRM) tests, the FAZ and the jam lines.

It is more traditional to focus validation activities on the jam lines; a scenario where the submarine has high forward speed compared to its rate of change of depth. This can be seen in Figure 2a that shows how the calculated flow angle (or hydrodynamic angle of attack) varies with pitch angle for a number of hydroplane jams over range of speeds and jam angles; the maximum flow angle does not tend to exceed a non-dimensional pitch angle of ± 1.0 in the example shown in Figure 2.

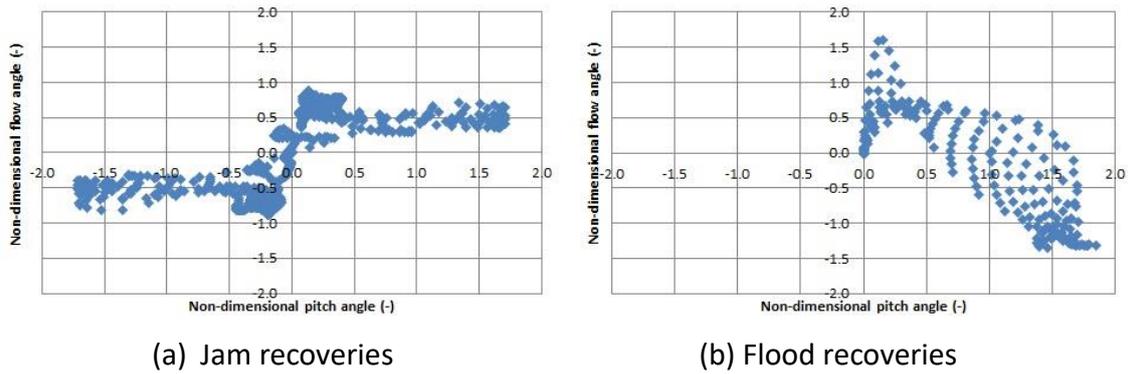


Figure 2: Pitch angle vs flow angle during jams and floods¹

When a submarine encounters a flooding incident, the flood water creates an out-of-trim condition which is then countered by increasing the speed of the submarine and if necessary blowing the Main Ballast Tanks (MBTs). These flood recovery manoeuvres can result in more significant flow angles on the submarine as a consequence of the low forward speed compared with the rate of change of depth and significant pitch on the submarine. It is likely that when the submarine is operating in an area that is adjacent to the FAZ boundary the flow angles experienced during a flood recovery would exceed those typically measured during the captive model tests.

Examples are shown in Figure 2b, that demonstrate how the calculated flow angles varies with pitch angle for a number of flood recovery manoeuvres for a range of different initial speeds, depths and flooding incidents. At the start of the flooding incident the submarine is heavy, sinking, and generates a pitch moment. This results in a rapid increase in the flow angle (in the positive direction) in excess of 1.5x the reference angle. As the submarine speeds up and gains hydrodynamic control over the flood mass, this reduces. However, as the MBTs are blown to create buoyancy, this results in the submarine ascending to the surface resulting in the flow angle becoming negative up to 1.5x the reference flow angle.

This implies that in order to fully validate the entire SOE boundaries, the mathematical model needs to be compared against measured responses that are typical of those around the jam lines and the FAZ. Furthermore, mathematical algorithms to account for the non-hydrodynamic aspects of the effects of flooding and blowing are also required in addition to the hydrodynamics model described earlier.

MODELLING FLOODING

Modelling floodwater entering a submerged submarine through a failed pressure hull penetration can be quantified with an adaptation of the Bernoulli equation for the velocity of fluid flow through a small orifice. The flow velocity through the hole is a function of the pressure inside the submarine, the hydrostatic pressure external to the submarine, the cross sectional area of the hole and a discharge coefficient.

If the total volume of flood water in the submarine compartment is significantly less than the total compartment volume, the pressure inside the submarine can be assumed to be

¹ Pitch and flow angles in Figure 2 have been non-dimensionalised using a typical maximum pitch angle that can be achieved during constrained model tests

at atmospheric pressure. In some cases (such as unsecured floods in small watertight compartments), it is more appropriate to take account of the rise in air pressure within the submarine.

The value of the discharge coefficient depends upon the type of flooding incident; for a direct penetration in the submarine pressure hull, a discharge coefficient of approximately 0.6, [9] to reflect the contraction of the flow through the flood hole. This can be considered appropriate for floods on a small diameter seawater system that is open to the sea, where the pressure hull fitting has failed. For systems that require larger pressure hull penetrations, it is normal to include some ability to secure the flood (through hull-mounted valves) should a failure occur. Therefore, for these types of securable systems, it is more likely that any failure is going to be within the pipework rather than the pressure hull penetration itself. In this case, a discharge coefficient of 0.6 may not be appropriate and may be somewhat less than 0.6 since there can be significant losses in the flow velocities due to the pipework system that the floodwater has to pass through.

Once the rate at which flood water enters the submarine is known, the impact of that flood water on the submarine response is required. The successful recovery of a submarine is sensitive to pitch angle, so the dynamic effect of the longitudinal centre of gravity of the flood mass may have a significant effect on the response of the submarine following a flood. Modelling this effect, taking account of the highly non-linear sloshing behaviour of the floodwater in the compartment would require complex CFD calculations to be coupled with a submarine manoeuvring simulation; this is not currently considered to be a practicable approach due to the significant computational cost associated with calculating the large number of manoeuvring trajectories required to define the FAZ. Therefore, it is considered appropriate to apply a quasi-steady state assumption for the flood water by taking the compartment geometry, and calculating the flood mass and its centroid for the instantaneous pitch angle.

MODELLING HIGH PRESSURE AIR BLOWING

Blowing high pressure air into MBTs can be divided into three parts: the flow of air from high pressure bottles, water flow out of from the ballast tank and the evolution of the pressure in the ballast tank, [9].

To model the flow of air from the bottle into the tank through a valve (that acts as a nozzle), [9] neglected pressure losses and heat transfer in the pipework that connected the bottle to the tank. A method was derived by [9] using a theory based on one dimensional steady flow of an ideal compressible gas. Since this method does not account for any pipework pressure losses, these algorithms are most suited to cases where the emergency bottle group is located adjacent to the blow nozzle in the MBT as any connecting pipework will be short. This would probably be the case where the HP air system has been designed to have a dedicated emergency blow bottle group, external to the pressure hull, which is independent from any normal HP air system, internal to the submarine, Figure 3. However, an alternative configuration is possible with the air from the emergency bottle group being supplemented with air from the main HP air system. To allow this arrangement to function, the pipework system is significantly more complex because the bottle groups supplying the main HP air system bottles (HPA) also have to supplement the emergency system, which has its own set of bottles, when required – see Figure 4. The other point to note is that the simplistic model is based on the assumption that the gas behaves as an ideal gas; [12],

suggested that more sophisticated engineering software simulation methods coupled with the assumption that air behaves as a real gas would be better suited to model the complexities of an HP air blow that includes a main ring main. A number of commercial engineering simulation codes are available to model pneumatic systems; AMESim, [11], and Flowmaster are two examples. QinetiQ have developed AMESim real gas models of representative HP air systems in submarines for blow modelling.

By using either the simplified blow model or the complex AMESIM model to determine the mass flow rate of air at the nozzle can be determined, and the pressure at the tank nozzle is then also known; it is the difference between the pressure in the tank and external water pressure that results in water being forced from the ballast tank through the flood grillages at the bottom of the MBT; [9] applied the Bernoulli equation at the ballast tank flood grillages to determine the volumetric flow of the water.

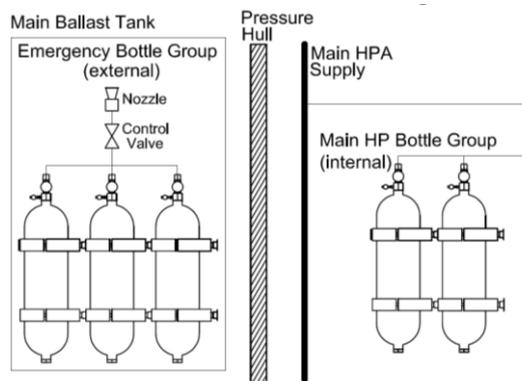


Figure 3: Arrangement for separate emergency HP air system

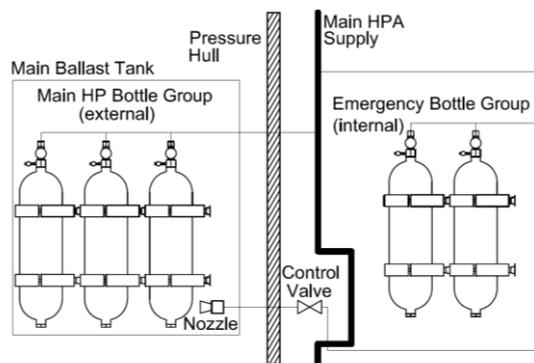


Figure 4: Arrangement for an integrated main and emergency HP air system

Once the flow of air to the ballast tank nozzle and the flow of water from the ballast tank are understood, the next step in the process is to understand how the air blown into the tank manifests itself as pressure. According to [9], the air is blown at high velocity, rapidly mixing with the water in the tank promoting good heat transfer from water to air (which will cause the air to expand); this process is considered isothermal and the ideal gas law can be used to determine the volume of the air in the tank. As the hydrostatic pressure decreases when the submarine drives towards the surface, the air volume within the MBT will expand. This results in an adiabatic process that decreases the air temperature in the MBT.

There is also an additional physical phenomenon to take into account during a flood recovery. During a recovery the submarine will adopt a positive pitch angle, which means that the forward MBTs are at a lower hydrostatic pressure than the aft ones. This phenomenon is considered to be significant when considering flood recoveries; whilst all MBTs may have equal masses of air blown into them, the air in the forward MBTs will have expanded to create a larger buoyant volume than the air in the aft ones. The impact of this is an additional pitch moment due to the differential expansion of air within the MBTs; this effect becomes more significant as the submarine approaches the surface.

FREE RUNNING SUBMARINE MODELS

A free-running model is a geosim of a full-scale submarine; it is used as a tool during the design and operation phases to gather data relating to the hydrodynamic manoeuvring and control performance of a submarine design. As part of the process for modelling the manoeuvring performance of a submarine, there are a number of reasons for undertaking FRM tests which include:

- Validation of the mathematical model;
- Manoeuvres suitable for System Identification, leading to improvements in the mathematical model predictions;
- To explore different control strategies;
- To investigate and validate the boundaries of the SOE
- Design of suitable motion control systems.

The UK's Submarine Research Model (SRM) capability has been employed in all the above areas, but is chiefly used for exploring the extremes of the manoeuvring envelope [4]. The SRM capability was first developed in the 1980's; it consists of an aluminium pressure hull that is 4.5m long with a diameter of 0.6m. Glass Reinforced Plastic (GRP) cladding is attached to this pressure hull to make it conform to the external shape of a range of submarine geometries with L/D ratios typical of SSNs or small SSKs, see [13]. The SRM is best suited to investigate hydroplane jams and subsequent recovery strategies, which are, by their nature, typically conducted at higher speeds. This first-generation design does not have active ballast control so is not particularly suited to slow speed operations, such as those required to investigate flood recovery scenarios.

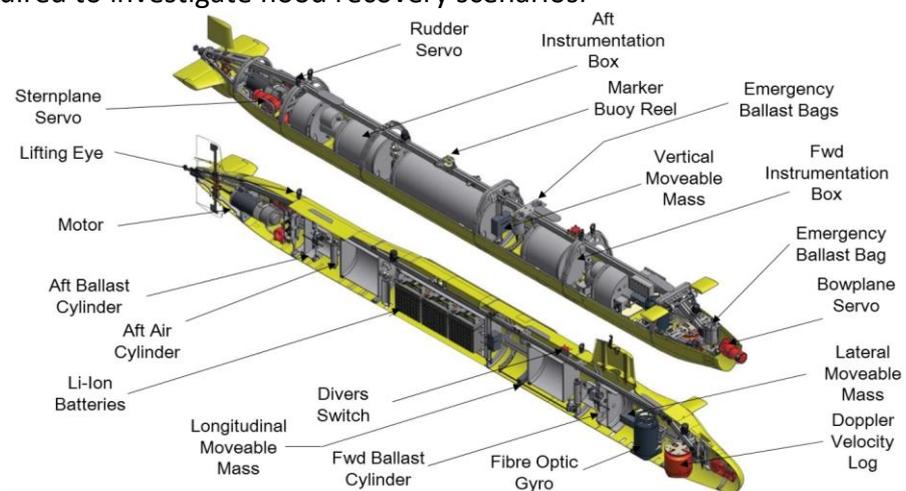


Figure 5: SRMII Free Running Model

In support of UK MOD studies to develop a replacement for the Vanguard Class SSBNs, this capability was upgraded, [13], to the Submarine Research Model II (SRMII) Figure 5. More detail about the design and operation of the SRMII can be found in [13] and [14], whilst the history of the development of FRM technology is discussed in [15]. The application of free-running models within the overall test and evaluation of a submarine's manoeuvring and control performance is detailed in [16], with further details as to why physical model experimentation of this type is still relevant explained in [17]. One of the most significant improvements in the SRMII design was the inclusion of an automated ballast and trim system that is described in the next section; this hugely capable system provides the means to investigate flood recoveries in submarines to inform on the validation of the entire SOE.

BALLAST AND TRIM SYSTEM OVERVIEW

The ballast system in the SRMII consists of two open ended cylinders, each containing an internal piston. The position of this piston is controlled by a stepper motor which changes the size of the 'dry' volume contained behind the piston. The capacity of each cylinder is just over 10 litres. The innovative approach to minimising the power requirement of the stepper motor was to keep the dry side (inside the cylinder) pressurised to offset the external hydrostatic pressure. The air is supplied from a small diver's air bottle inside the model. A regulator on the air supply line to the ballast cylinder ensures that the pressure behind the piston is maintained only slightly higher than the ambient hydrostatic pressure. The ballast cylinders are located at either end of the model and can be operated independently to provide a wide range of mass and moment changes; the components, assembly and locations within the model are shown in Figure 6.

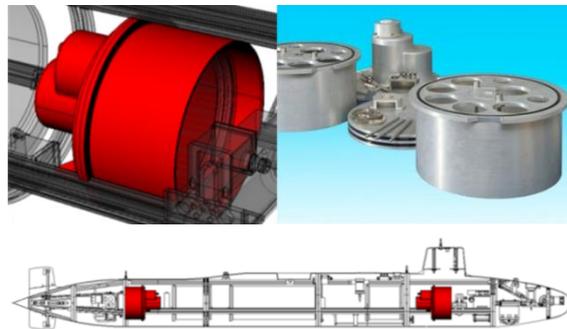


Figure 6: Ballast system components in SRMII

The design also incorporates a static trim system, consisting of three movable masses, each fixed to a lead screw, and driven by stepper motors. These align with the principal axes of the model such that the longitudinal mass can generate pitch moment, the transverse mass can generate roll moment, and the vertical mass changes the height of the centre of gravity (CoG) of the model. As a combined ballast and trim system the model can be either programmed with predetermined changes to the ballast and trim condition, or for the on-board software to order changes autonomously as required.

In the context of modelling flood recoveries, the ballast and trim system can replicate the change in mass and moment imparted on the submarine for a range of significant flood scenarios. This can be done in a number of ways, potentially by embedding the mathematical representation of the flood and blow algorithms in the on-board software as a closed loop system responding to the instantaneous depth and pitch of the model.

Alternatively the scenario can be treated as open loop, imparting a known change in mass and moment on the model. To provide validation evidence of the hydrodynamic models in a flow regime that is typical of flood recoveries, the most appropriate solution is to treat the scenario as open loop and use simulation to generate *a priori* time histories of changes in mass and moment, which are representative of a flood recovery. It was this approach that was adopted during the experiments described following section.

EXPERIMENT RESULTS

Flood and blow trajectories

The ballast and trim system was used to provide systematic changes in mass and moment, representative of different flood and blow scenarios, to achieve a wide range of incident flow angles (both positive and negative) for mathematical model validation. The flood scenarios replicated floods of increasing severity at a typical aft engine room location; where the size and complexity of seawater systems used are more likely to result in a significant flooding incident, should a breach to the watertight integrity occur. For the blow scenarios, the ballast and trim system was used to represent blowing of HP air into a forward MBT that made the model buoyant and created a bow up pitch.

For each flood scenario, the aft ballast cylinder was used to increase the mass of the model, whilst the forward cylinder was adjusted to ensure a representative moment. Once the flood had been represented, there was a short delay to replicate the time taken for the crew to react and execute their emergency recovery procedures. Following this time delay, the appropriate propulsor response was applied and the model was driven to the surface using a specifically designed autopilot that controlled pitch during the ascent.

All of the flood scenarios were initiated whilst the model was at low forward speed, and therefore the change in mass and moment created by the flood induced a significant pitch angle. Accelerating the model enables the hydroplanes to partially counter the flood mass and moment and reduce the pitch angle to an acceptable level. By varying the initial speed of the model and the recovery RPM, a range of peak pitch angles (the point at which the hydroplanes were able to reduce pitch) were obtained.

For each blow scenario, the forward ballast cylinder was used to make the model light, whilst the aft cylinder was adjusted to ensure a representative moment. Once the change in trim condition of the model had been achieved, the response of the model (in terms of demanded RPM and autopilot actions) was the same as that for the floods. Again, by varying the initial speed and the recovery RPM, a range of peak pitch angles could be obtained. Varying the initial conditions also enabled the model to undergo a range of hydrodynamic angles of attack (both positive and negative); comparisons of simulations with the experimental trajectories allows a greater level of assessment of the validity of the mathematical model for these type of manoeuvres.

Figure 7 shows typical flood and blow recovery sequences, taken from video stills, of the model during the run-up period (a), at the initiation of the flood or blow (b), and as the model drives to the surface (c). For the flood case, a large amount of air can be seen being vented from the aft ballast cylinder; for the initiation of the blow scenario, since air is added to the forward cylinder, it is only venting a small amount of air from the aft cylinder to represent the required pitch moment. The time sequence of the stills is the same for the

flood and the blow so the figure shows clearly the model reaching the surface quicker for the blow case.

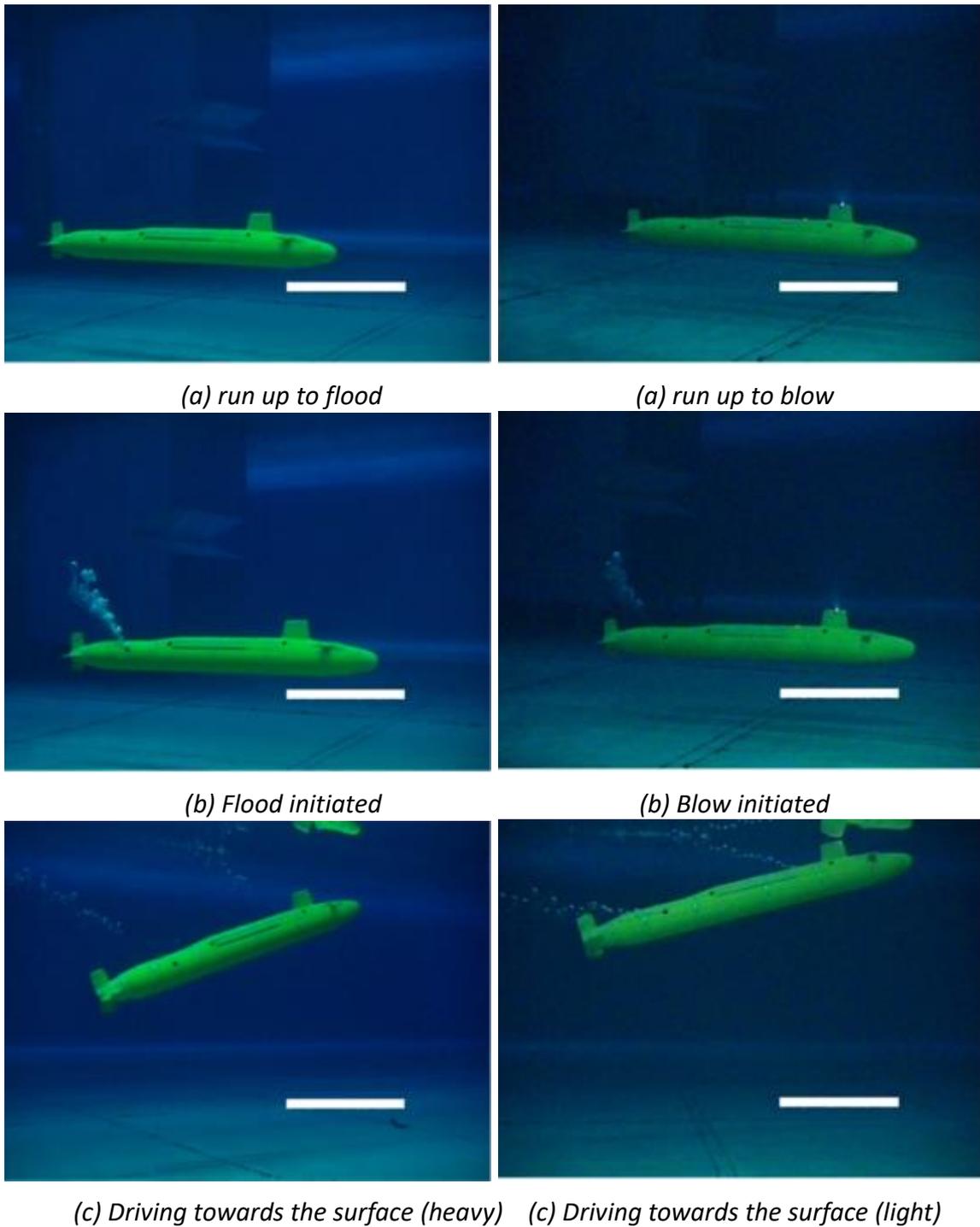


Figure 7: Images of flooding and blowing

Figure 8 provides example measurements of flood and blow responses of the model (blue lines are the measured responses, orange are the demanded responses) compared with simulation (in this case red lines are the predicted responses and green are the demanded). In both cases, the predictions of the initial response in depth and pitch compare well with measurements, as do the ascent parts of the trajectory as the model drives to the surface.

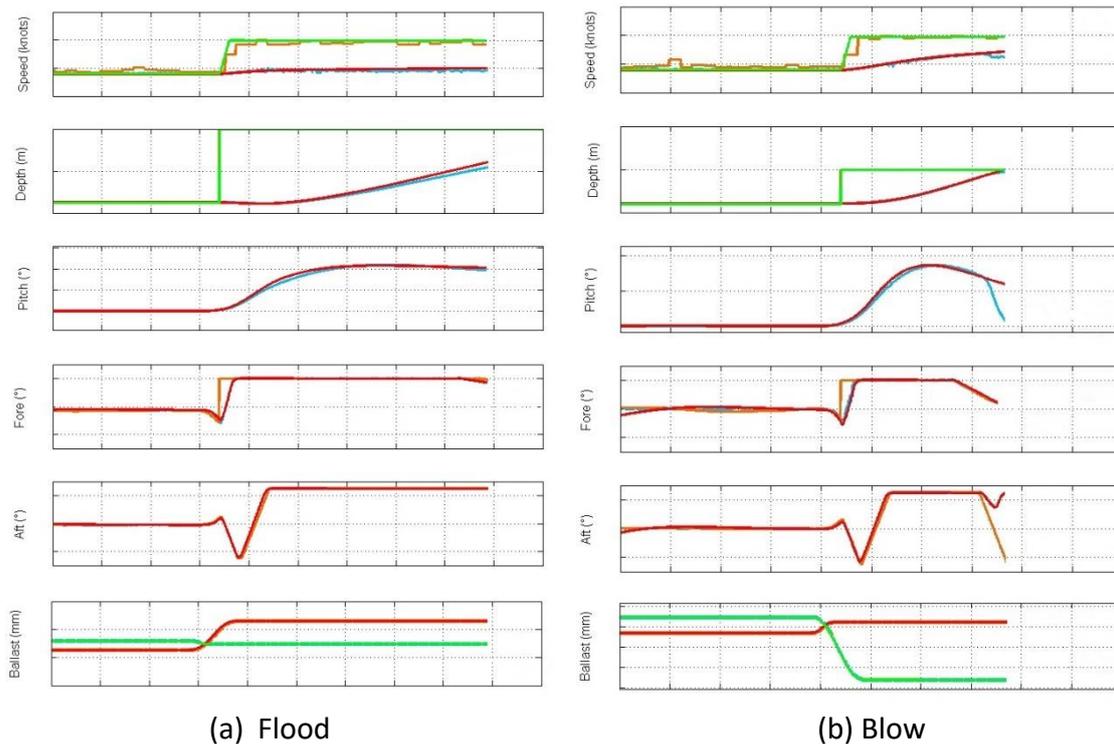


Figure 8: Trajectories of flooding and blowing

Initial exploratory tests were conducted in QinetiQ’s Ocean Basin at Haslar, where the water depth is around one model length. In order to allow the recovery trajectories to fully develop for the more extreme flood and blow cases, a deeper initial depth is required, so similar tests are planned for the deep-water reservoir facility.

Simulation correlation

To provide an overall view of the quality of the simulations when compared with the experiments, a correlation of the simulated peak pitch angles from the flood and blow trajectories with the measurements is shown in Figure 9. This is a similar concept to that used when assessing simulation of peak pitch angles during hydroplane jam recovery scenarios.

The results of the correlation show that predictions of maximum pitch angle for both (a) flood and (b) blow scenarios correlate very well with the measured pitch angle. However, also shown is a similar correlation plot for jam scenarios (c) which shows less scatter in the data compared with plots (a) and (b). This suggests either some limitations in the mathematical model, or an illustration of the increased level of uncertainty in conducting these types of experiments compared with hydroplane jam recovery manoeuvres.

To illustrate the extent to which the mathematical model has been validated, in the context of the FAZ, the combinations of the measured flow and pitch angles for the flood and blow scenarios generated during the experiment are shown in Figure 10. Also shown, for comparison, is the area indicated within Figure 2(b) as the flow and pitch angle combinations that are generated when calculating a FAZ boundary.

This shows that the extremes of flow angle that are predicted to occur were not sufficiently represented during the experiments and where within the range of the captive model tests. This is largely due to the inability to test the necessary combinations of floods and blows in

the limited depth of the Ocean Basin. It is possible that it is these extreme flow angles where the accuracy of the mathematical model may be called into question.

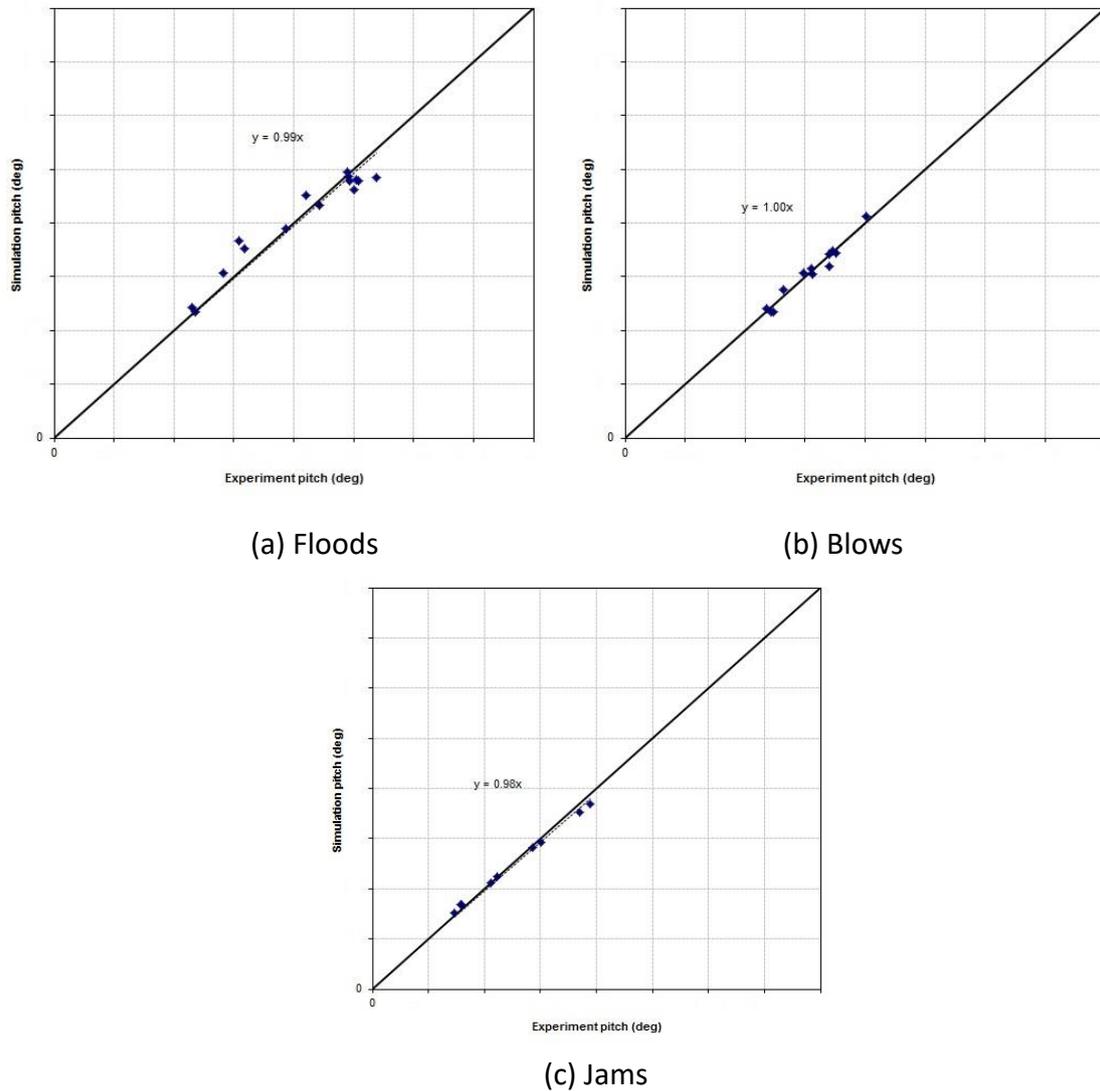


Figure 9: Correlation of simulated peak pitch angles with measurements

Once the more extreme tests of combinations of flood and blows have been undertaken, then the full validation of the mathematical model can be performed. If the model requires improvements then this could be done by including higher order terms in the Gertler and Hagen mathematical expressions such as [18], who augmented the force and moment equations with up to fifth order terms derived from wind tunnel experiments on a captive model. However, the example data shown in [18] indicates that this approach does not take into account stall effects. In reality, high flow angles at low speeds are likely to result in stall and this may require a different approach. One alternative approach for capturing the non-linear effects due to high angles of incidence may be to introduce look-up tables that are derived from captive models tests. These look-up tables would be accessed at each time step of the simulation to provide a force and moment rather than through the coefficient based approach. One disadvantage of this approach is that the lookup table must capture the range of data required for simulation, as typically look-up tables cannot be used for extrapolation, whereas the coefficient based approach can extrapolate (with due caution) for scenarios outside of the fitted range of data.

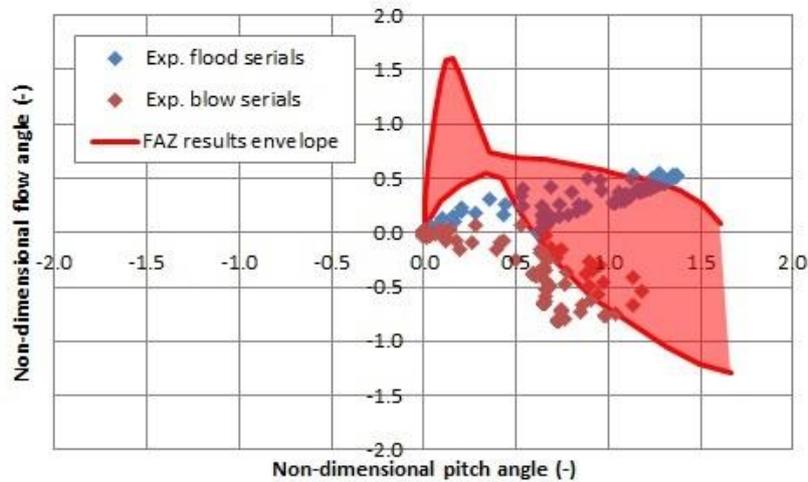


Figure 10: Hydrodynamic angle of attack vs pitch angle compared to FAZ

CONCLUDING DISCUSSIONS

This paper has discussed one specific application of a free running submarine model capability; to explore and validate responses that are typical of a submarine recovering from a flood or responding to a blow. The experiments successfully proved the methodology for conduct and analysis of validating trajectories for flood recoveries using QinetiQ's free running model capability.

The correlation that has been undertaken with the current data has shown that the predicted peak pitch angles compare well with those measured during the experiment. However, as a consequence of some of the limitations with this experiment, the flow angles achieved were not as extreme as those predicted to occur during flood scenarios around the FAZ boundary and moreover, were within the range over which a captive model would be tested (from which the hydrodynamic coefficients are derived). Therefore, to some extent the correlation was expected to be good.

Further tests will be planned using a deep-water test facility (reservoir) to provide the additional water space that would allow tests to be conducted at deeper initial depths. These conditions would generate more significant flow angles on the model thus providing validation data in these more extreme but still pertinent, areas.

These tests provide confidence that these validated tools are providing an understanding of the behaviour of a deeply submerged submarine that is subjected to a major flood as a consequence of a failure in a pressure hull penetration. This capability underpins the provision of safe operating envelopes of current and future underwater platforms during normal operations and emergency conditions which is a key requirement for any nation that operates submarines with due governance regarding safety.

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